Circular CA-01: Calculating G-Gradients for 2300AD

G-gradient values can be derived from Newton's Law of Gravitation, equation (1)

Equation (1) $$g = \frac{GM}{r^2}$$

…where

- g = standard gravity of 9.80665 m s^{-2}
- G = Gravitation Constant of 6.67428×10^{-11} m^3 kg^{-1} s^{-2}
- M = Mass, kg
- r = radius, m

We are interested in the distance (radius) from the body's core at which a particular g-gradient occurs. Thus, equation (1) can be re-written in terms of *r*, as expressed in equation (2).

Equation (2) $$r = \left(\frac{GM}{g}\right)^{1/2}$$

Equation (2) will provide the distance (radius) at which the g-gradient is 1.0. However, the two g-gradients of interest in ***2300AD*** are the 0.0001 and 0.1 ones. Let us modify equation (2) to reflect these gravity levels, as shown in equation (3) and (4).

Equation (3) $$r_{0.0001grad} = \left(\frac{GM}{0.0001g}\right)^{1/2}$$

Equation (4) $$r_{0.1grad} = \left(\frac{GM}{0.1g}\right)^{1/2}$$

For planetary bodies, it is convenient to express mass as a value of the mass of the Earth, where Earth's mass is 5.9742×10^{24} kg.

Let's simplify equation (3), as shown in equations (5.1) and (5.2)

Equation (5.1) $$r_{0.0001grad} = \left(\frac{\dfrac{6.67428\times10^{-11}\,m^3}{kg\,s^2}\times 5.9742\times10^{24}\,kg\,M}{\dfrac{9.80665\,m}{s^2}\times 0.0001} \right)^{1/2}$$

Equation (5.2) $r_{0.0001grad} = 6.37649\times10^{9}\,m\,M^{1/2}$

The distance in equation (5.2) can be expressed in kilometers, as shown in equation (6), or astronomic units (au) in equations (7.1) and (7.2), where there is 149.5×10^6 km in an au.

Equation (6) $r_{0.0001grad} = 6.377\times10^{6}\,km\,M^{1/2}$

Equation (7.1) $$r_{0.0001grad} = \left(6.37649\times10^{6}\,km\,M^{1/2}\right)\times\left(\frac{1\,au}{149.5\times10^{6}\,km}\right)$$

Equation (7.2) $r_{0.0001grad} = 4.265\times10^{-2}\,au\,M^{1/2}$

Equation (4) can be treated in a similar manner as equation (3) to yield equations (8.1), (8.2) and (8.3).

Equation (8.1) $$r_{0.1grad} = \left(\frac{\dfrac{6.67428\times10^{-11}\,m^3}{kg\,s^2}\times 5.9742\times10^{24}\,kg\,M}{\dfrac{9.80665\,m}{s^2}\times 0.1} \right)^{1/2} \times \frac{1\,km}{1000\,m}$$

Equation (8.2) $r_{0.1grad} = 2.016\times10^{4}\,km\,M^{1/2}$

Equation (8.3) $r_{0.1grad} = 1.349\times10^{-4}\,au\,M^{1/2}$

If we look at stars, it becomes more practical to express mass in terms of the mass of our sun, Sol, which is 1.98892×10^{30} kg. It is also more useful to express distance in au.

Equation (3) can then be re-written as equations (9.1) and (9.2).

Equation (9.1) $$r_{0.0001grad} = \left(\frac{\frac{6.67428 \times 10^{-11}\, m^3}{kg\, s^2} \times 1.98892 \times 10^{30}\, kg\, M}{\frac{9.80665\, m}{s^2} \times 0.0001} \right)^{\frac{1}{2}} \times \frac{1\, au}{149.5 \times 10^9\, m}$$

Equation (9.2) $$\boxed{r_{0.0001grad} = 2.4610\, au\, M^{\frac{1}{2}}}$$

Equation (4) can be re-written as equations (10.1) and (10.2).

Equation (10.1) $$r_{0.1grad} = \left(\frac{\frac{6.67428 \times 10^{-11}\, m^3}{kg\, s^2} \times 1.98892 \times 10^{30}\, kg\, M}{\frac{9.80665\, m}{s^2} \times 0.1} \right)^{\frac{1}{2}} \times \frac{1\, au}{149.5 \times 10^9\, m}$$

Equation (10.2) $$\boxed{r_{0.1grad} = 0.0778\, au\, M^{\frac{1}{2}}}$$